# Determination of the Photon Force and Power

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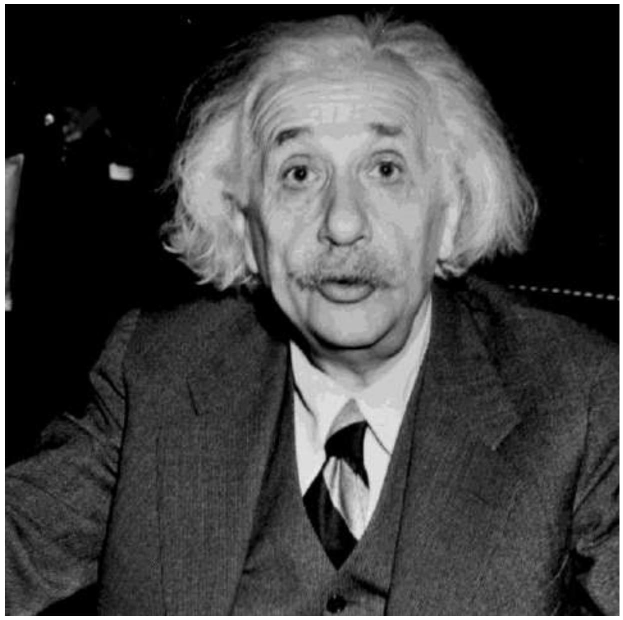
## Abstract

Photons, although lacking in mass, do have momentum — so when they hit a particle like an electron, they apply a force onto it. How much force does photon exert on the particle with which it interacts? It's a problem that's confounded physicists for nearly 150 years − and it may now have a solution. In this article, we derive the photon power and force of the electromagnetic radiation photon on material particles by a direct application of the quantum theory of electromagnetic radiation. In this derivation, Planck's quantum hypothesis, the Newtonian mechanics, the Einsteinian relativity, and the law of conservation play central roles.



**Albert Einstein** was a German-born theoretical physicist, widely acknowledged to be one of the greatest physicists of all time. Einstein is known for developing the theory of relativity, but he also made important contributions to the development of the theory of quantum mechanics.

In empty space, the photon − the basic unit of all light − moves at c (the speed of light) and its energy and momentum are related by $E = pc$, where p is the momentum of the photon. This derives from the following relativistic relation, with $m_0 = 0$:

$$E^2 = p^2c^2 + m_0^2c^4.$$

In some situations, photon behaves like a wave, while in others, it behaves like a particle. The photons can be thought of as both waves and particles. In 1924 a French physicist **Louis de Broglie** developed a formula to relate this dual wave and particle behavior:

$$E = h\upsilon, \quad c = \lambda\upsilon, \quad E = \frac{hc}{\lambda} = mc^2,$$

where E and m are the energy and mass of the photon, υ and λ are the frequency and wavelength of the photon, h is the Planck constant, c is the speed of light. From this we obtain the definition of the photon wavelength through the Planck constant and the momentum of the photon:

$$\lambda = \frac{h}{mc}$$

This equation is used to describe the wave properties of matter, specifically, the wave nature of the electron:

$$\lambda_e = \frac{h}{m_e v}$$

where $\lambda_e$ is wavelength, h is Planck's constant, $m_e = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ is the relativistic mass of the electron, moving at a velocity v.

$$p_e = \frac{h}{\lambda_e}$$

From this it follows that,

$$\frac{dp_e}{dt} = -\frac{d\lambda_e}{dt} \times \frac{h}{\lambda_e^2}$$

$$\frac{dp_e}{dt} = \frac{p_e^2}{h} \times -\frac{d\lambda_e}{dt}$$

Sir Isaac Newton first presented his three laws of motion in the "**Principia Mathematica Philosophiae Naturalis**" in 1686. His second law defines a force exerted by a photon on the electron to be equal to the rate of change in momentum of the electron:

$$F = \frac{dp_e}{dt}$$

$$\boxed{F = \frac{p_e^2}{h} \times -\frac{d\lambda_e}{dt}}$$

The work **W** done by the photon force on the electron equals the change in the electron's kinetic energy $\mathbf{E_k}$:

$$W = dE_k = F\,dx$$

$$\frac{dE_k}{dt} = F\,v = \frac{p_e^2\,v}{h} \times -\frac{d\lambda_e}{dt}$$

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When a photon with energy **E** collides with an electron associated with the energy $\mathbf{E}_{\boldsymbol{e}}$. Some of the energy and momentum is transferred to the electron, but both energy and momentum are conserved in this elastic collision. After the collision the photon has energy **E'** and the electron has acquired an energy $\mathbf{E}_{\boldsymbol{e}}' > \mathbf{E}_{\boldsymbol{e}}$. One of the most powerful laws of physics is the **Law of conservation of energy**. For a collision occurring between photon and particle in an isolated system, the total energy of the two particles before the collision is equal to the total energy of the two particles after the collision. That is,

$$\mathbf{E + E_{\boldsymbol{e}} = E' + E_{\boldsymbol{e}}'}$$
$$\mathbf{-(E' - E) = (E_{\boldsymbol{e}}' - E_{\boldsymbol{e}})}$$
$$\mathbf{-(E' - E)} = m_0c^2 + \mathbf{E_k'} - \mathbf{(}m_0c^2 + \mathbf{E_k)}$$
$$\mathbf{-(E' - E) = (E_k' - E_k)}$$

$$\mathbf{-\Delta E = \Delta E_k}$$

where $\mathbf{\Delta E_k}$ = change in kinetic energy of the electron and $\mathbf{\Delta E}$ = change in energy of the photon.

For an infinitely small change in the energy, the equation takes the form:

$$\mathbf{-dE = dE_k}$$

$$-\frac{dE}{dt} = \frac{dE_k}{dt} = \frac{p_e^2\,v}{h} \times -\frac{d\lambda_e}{dt}$$

Given that the power of the photon is the rate of energy loss of the photon:

$$P = -\frac{dE}{dt}$$

$$\boxed{P = \frac{p_e^2\, v}{h} \times -\frac{d\lambda_e}{dt}}$$

Thus, we have the formula for the calculation of the photon power and force of the electromagnetic radiation photon on material particles.

**Discussion:**

According to the law that nothing may travel faster than the speed of light – i.e., according to the Albert Einstein's law of variation of mass with velocity (the most famous formula in the world. In the minds of hundreds of millions of people it is firmly associated with the menace of atomic weapons. Millions perceive it as a symbol of relativity theory):

$$m_e = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_e^2c^2 - m_e^2v^2 = m_0^2c^2$$

That the electron's mass $\mathbf{m_e}$ in motion at speed $\mathbf{v}$ is the mass $\mathbf{m_0}$ at rest divided by the factor $\sqrt{1-\frac{v^2}{c^2}}$ implies: the mass of the electron is not constant; it varies with changes in its velocity.

Differentiating the above equation, we get:

$$m_e v\, dv + v^2 dm_e = c^2 dm_e$$

$$dm_e\,(c^2 - v^2) = m_e v\, dv$$

In relativistic mechanics (the arguably most famous cult of modern physics, which has a highly interesting history which dates back mainly to Albert Einstein and may be a little earlier to H. Poincaré), we define the energy $\mathbf{m_e c^2}$ which a moving electron possess to be = $m_0c^2 + E_k$.

$$\mathbf{m_e c^2} = m_0c^2 + E_k$$

$$\frac{dm_e c^2}{dt} = \frac{dE_k}{dt} = Fv$$

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$$\boxed{F = \frac{dp_e}{dt} = \frac{d(m_e v)}{dt}}$$

$$\mathbf{F} = \frac{dm_e}{dt} v + \frac{dv}{dt} m_e$$

$$\mathbf{F} = \mathbf{F}\frac{v^2}{c^2} + m_e a$$

$$\mathbf{F} = \frac{\mathbf{m_e a}}{1 - \frac{v^2}{c^2}}$$

So as v approaches c, the bottom term approaches zero and therefore the force approaches infinity. It requires an infinite amount of force to accelerate the electron to the speed of light. Because:

$$m_e = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Therefore:

$$F = \frac{m_e^3 a}{m_0^2}$$

For non-relativistic case ($v << c$), the above equation reduces to $F = m_0 a$.

$$\frac{p_e^2}{h} \times -\frac{d\lambda_e}{dt} = \frac{m_e^3 a}{m_0^2}$$

From this it follows that,

$$a = \frac{m_0^2 v^2}{h m_e} \times - \frac{d\lambda_e}{dt}$$

Thus, we have the formula for the calculation of the acceleration of the electron. For nonrelativistic case (v << c), the above equation reduces to:

$$a = \frac{m_0 v^2}{h} \times - \frac{d\lambda_e}{dt}$$

In no experiment, matter exists both as a particle and as a wave simultaneously. It is either the one or the other aspect.

$$\text{Radiation Pressure} = \frac{2 \times \text{Intensity}}{\text{speed of light in vacuum}}$$ **(Radiation pressure when a wave is 100% reflected)**

$$\text{Radiation Pressure} = \frac{\text{Intensity}}{\text{speed of light in vacuum}}$$ **(Radiation pressure when a wave is 100% absorbed)**

"It was an act of desperation. For six years I had struggled with the blackbody theory. I knew the problem was fundamental and I knew the answer. I had to find a theoretical explanation at any cost, except for the inviolability of the two laws of thermodynamics."

**− Max Planck**

"The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote.... Our future discoveries must be looked for in the sixth place of decimals."

**– Albert A. Michelson, 1894**

| **Classical Picture** | **Quantum Picture** |
|---|---|
| Energy of EM wave ~ (Amplitude) $^2$ | Energy of photon = $\frac{hc}{\lambda}$ |

The gravitational attraction between two electrons is given by:

$$F_G = \frac{Gm_e^2}{r^2}$$

The Coulomb repulsion between two electrons is given by:

$$F_E = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$\frac{F_E}{F_G} = 4.17 \times 10^{42}$$

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = 4.17 \times 10^{42} \times \frac{Gm_e^2}{r^2}$$

$$\frac{e^2}{4\pi\varepsilon_0 m_e c^2} = 4.17 \times 10^{42} \times \frac{Gm_e}{c^2}$$

| |
|---|
| Electron classical radius = 2.085 × Schwarzschild radius of electron |

The power radiated from a black body is proportional to its temperature, raised to the fourth power:

$$P = \varepsilon\sigma T^4 A$$

$$\frac{4P}{3c} = \frac{4\sigma T^4}{3c} \times \varepsilon A = \text{Radiation pressure} \times \varepsilon A$$

$$\boxed{P = \frac{3}{4} \times \varepsilon \times \text{Radiation force} \times c}$$

**References:**


- Light – The Physics of the Photon, by Ole Keller.
- **University Physics** with Modern Physics by Hugh D. Young.
- Isaac Newton and the Laws of Motion by **Andrea Gianopoulos.**